Stochastic Processes and Calculus

An Elementary Introduction with Applications

Uwe Hassler

Springer 2016

Teaching Material

The following figures are from the above book. They are provided to help instructors and students and may be used for teaching purposes as long as a reference to the book is given in class.

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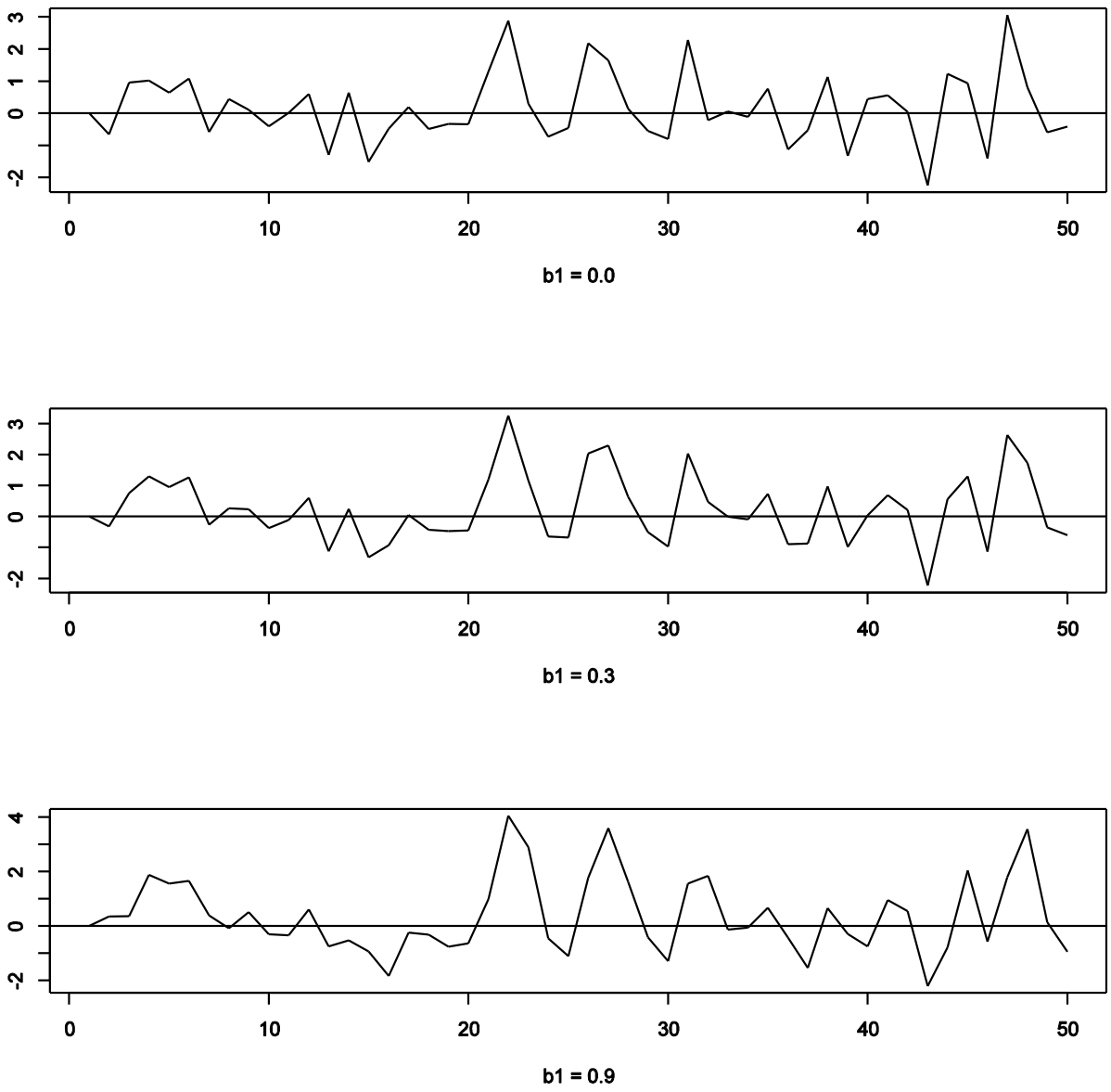

Figure 3.1 Simulated MA(1) Processes with $\sigma^2 = 1$ and $\mu = 0$

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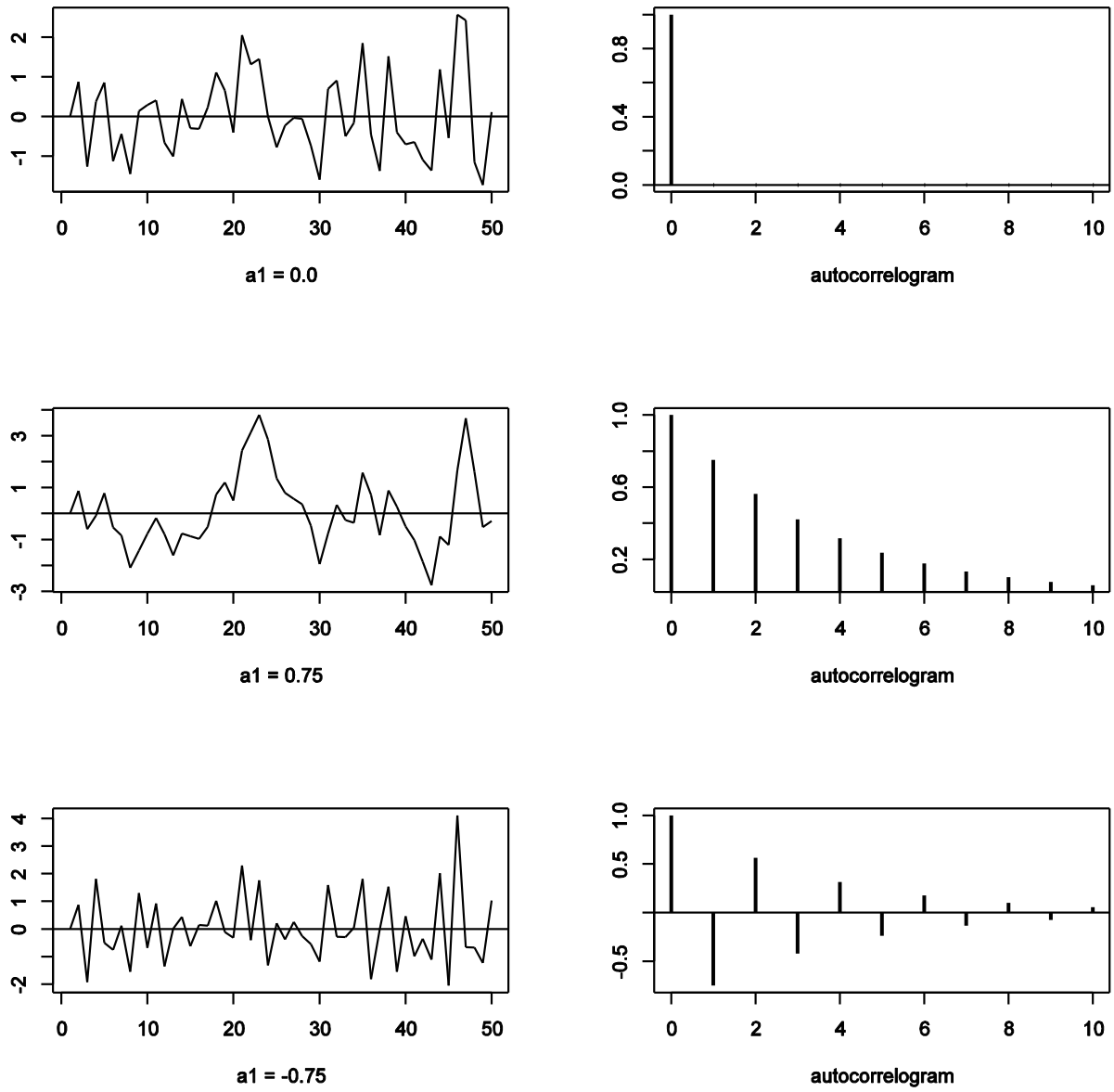

Figure 3.2 Simulated AR(1) Processes with $\sigma^2 = 1$ and $\mu = 0$

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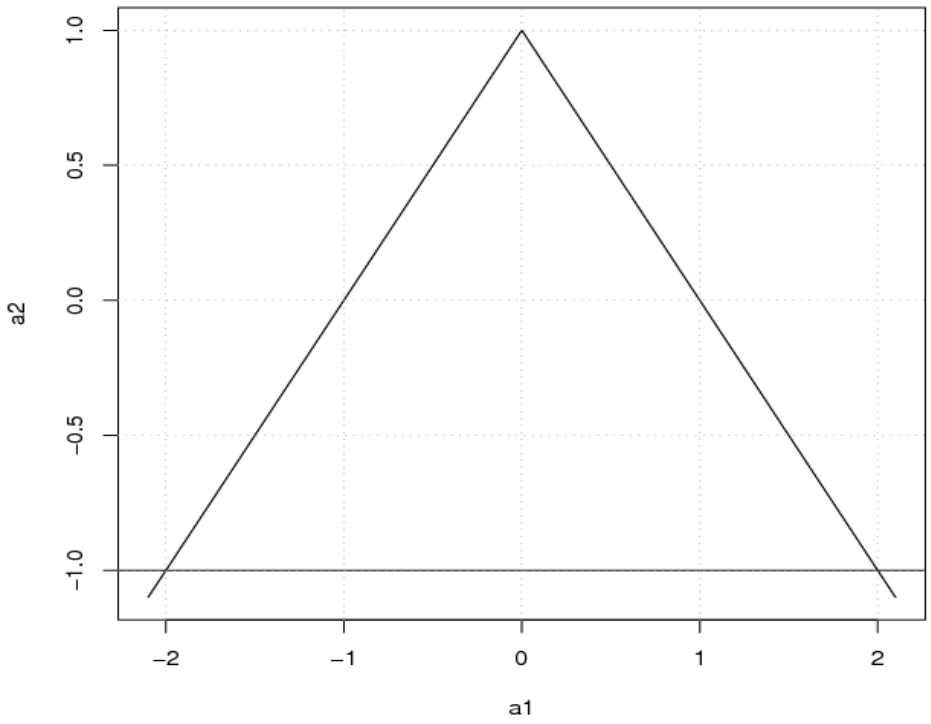

Figure 3.3 Stationarity Triangle for AR(2) Processes

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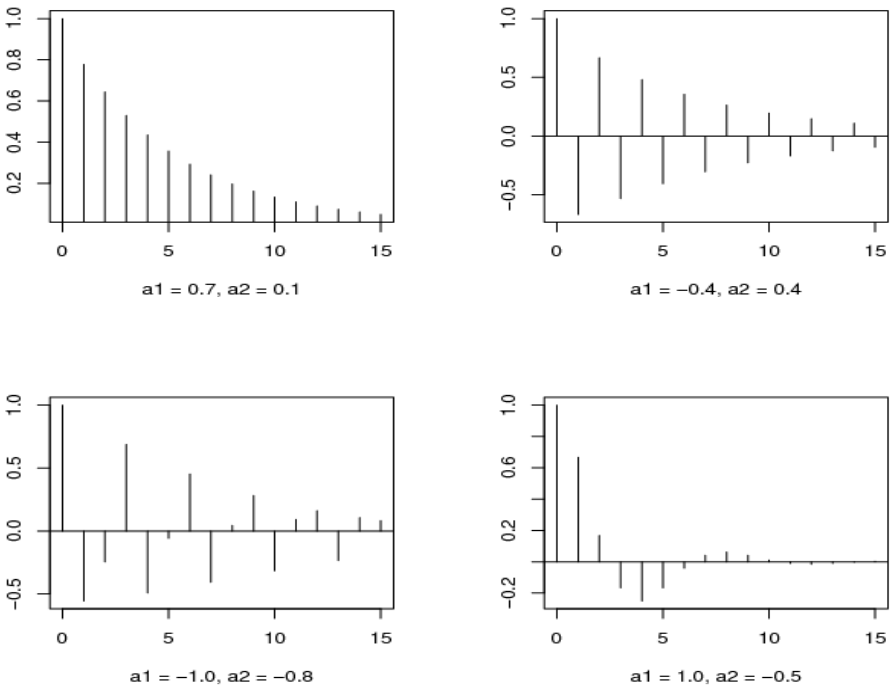

Figure 3.4 Autocorrelograms for AR(2) Processes

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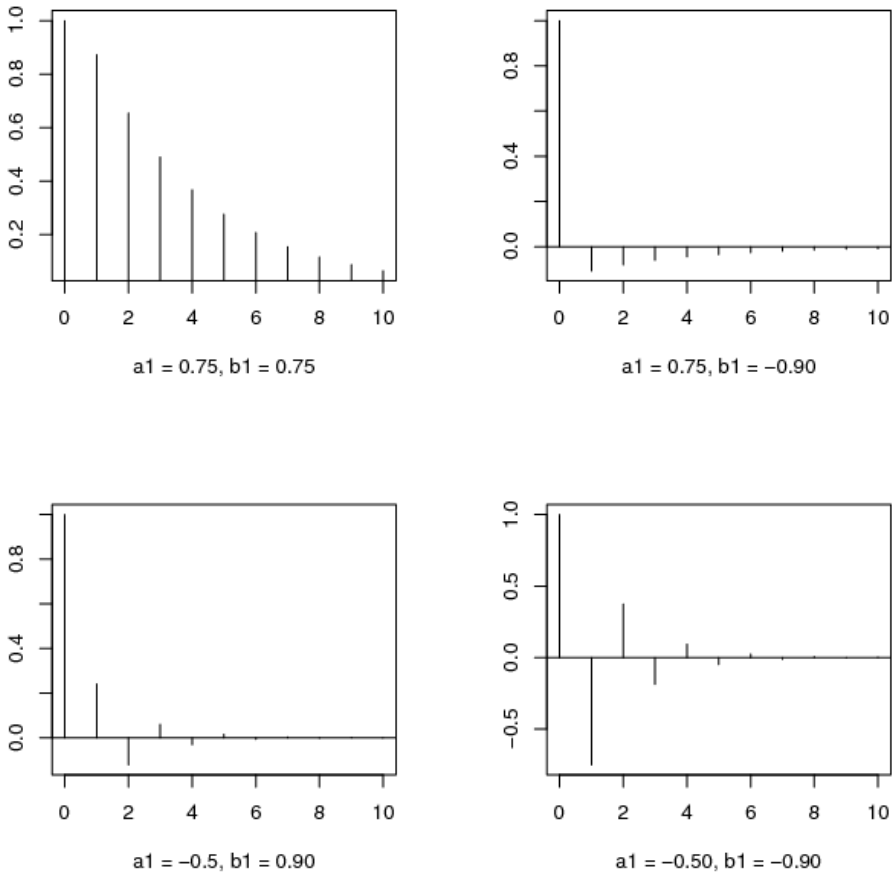

Figure 3.5 Autocorrelograms for ARMA(1,1) Processes

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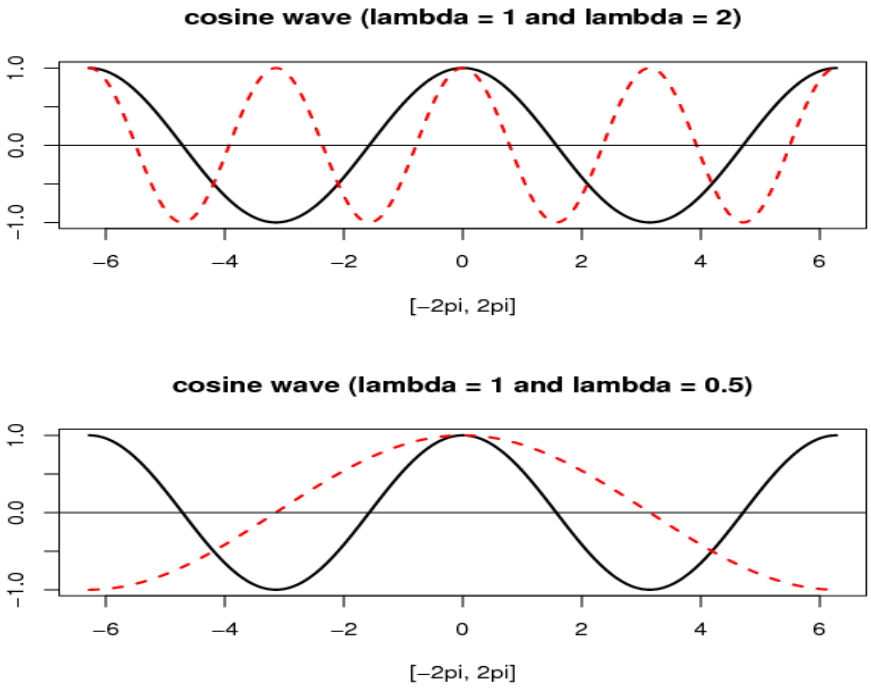

Figure 4.1 Cosine Cycle with Different Frequencies

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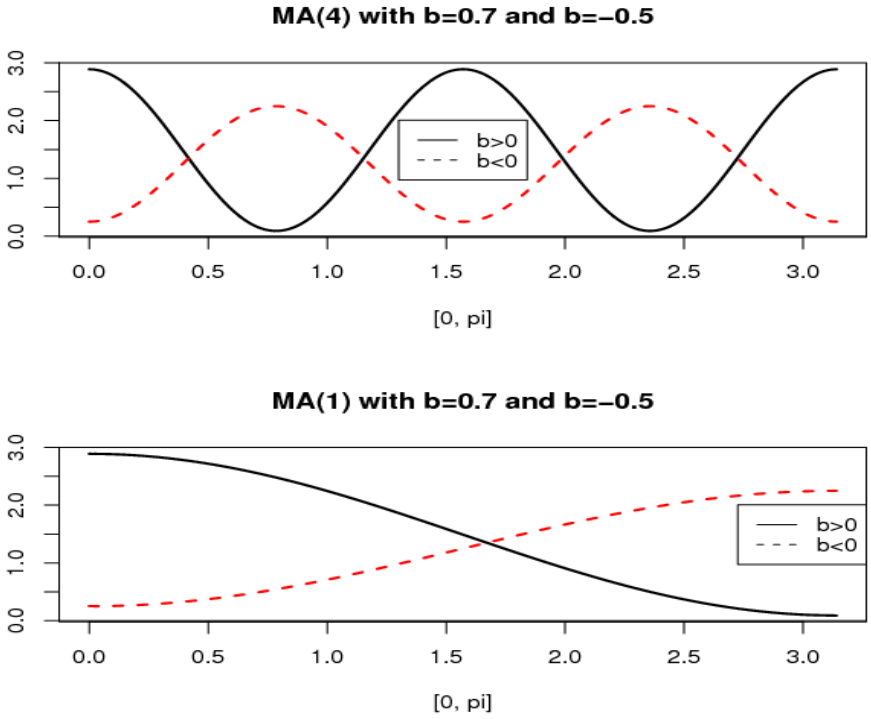

Figure 4.2 Spectra ($2\pi f(\lambda)$) of the MA(S) process from Ex. 4.2

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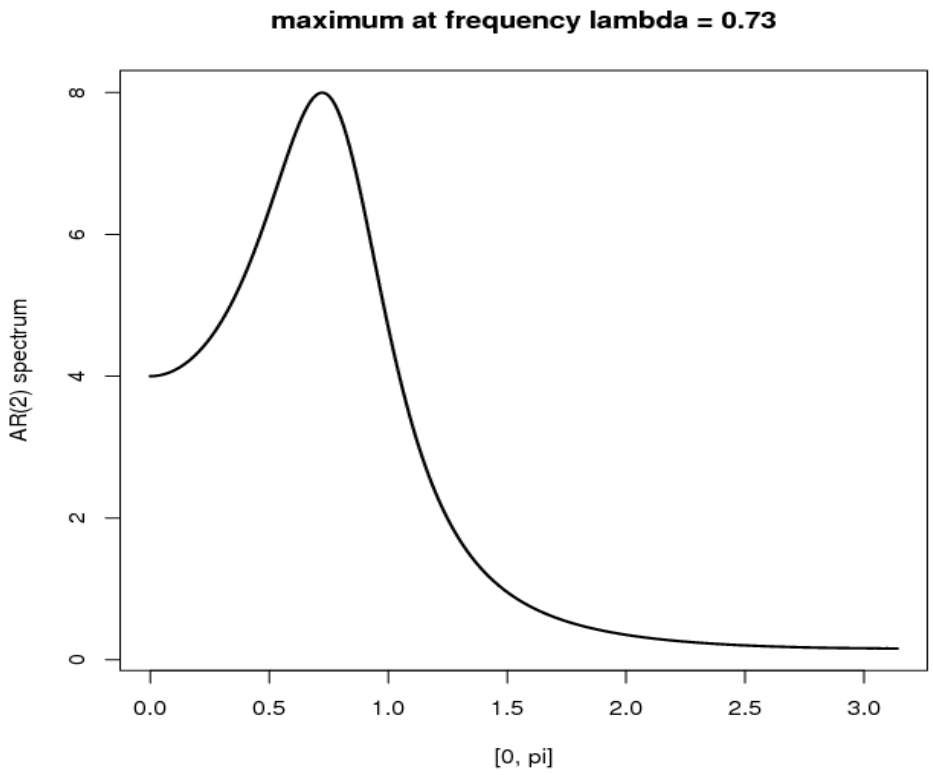

Figure 4.3 Spectrum ($2\pi f(\lambda)$) of Business Cycle with a Period of 8.6 years

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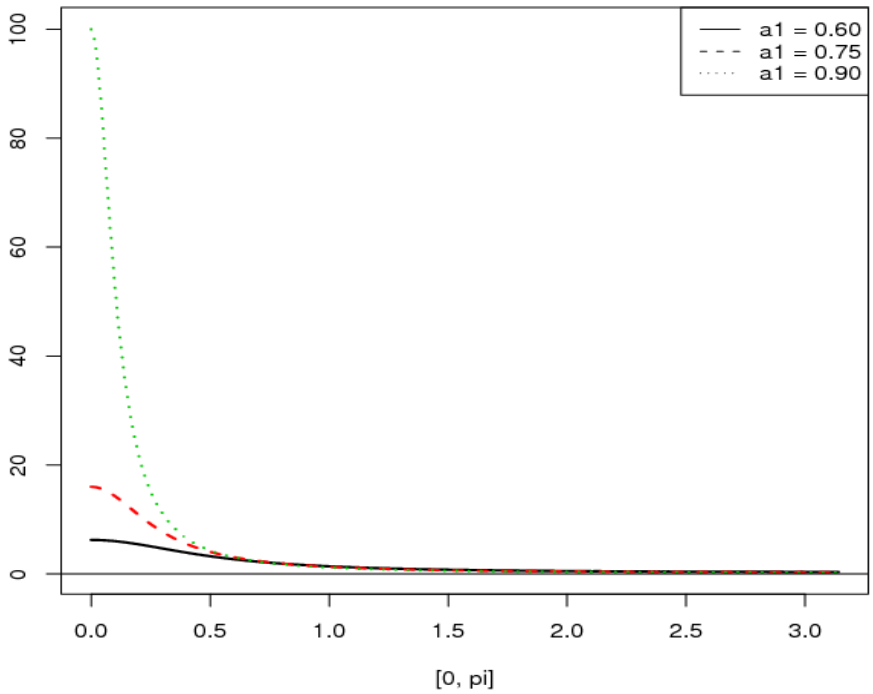

Figure 4.4 AR(1) Spectra ($2\pi f(\lambda)$) with Positive Autocorrelation

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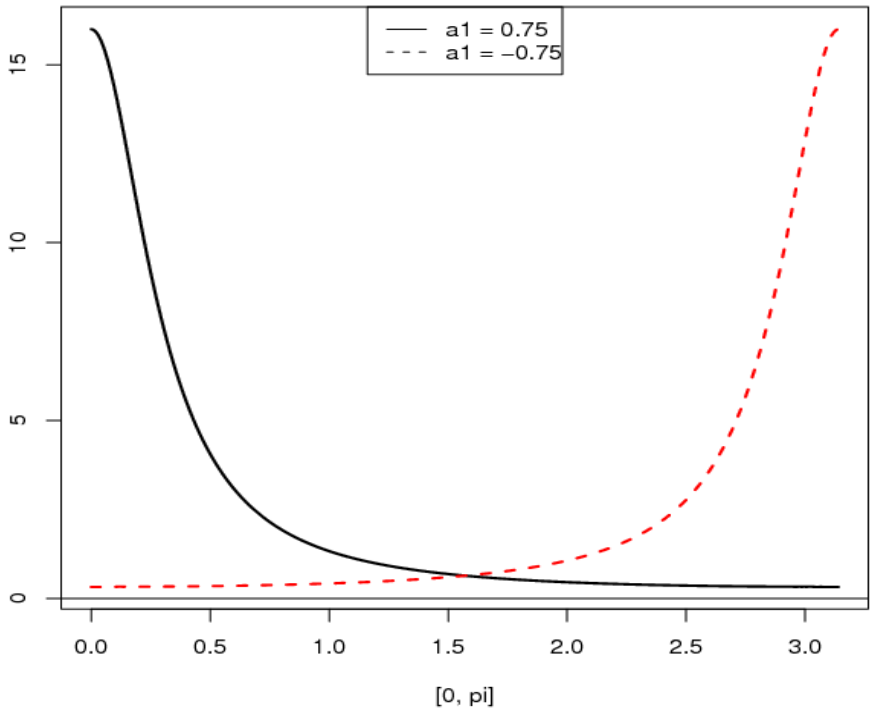

Figure 4.5 AR(1) Spectra ($2\pi f(\lambda)$), cf. Figure 3.2

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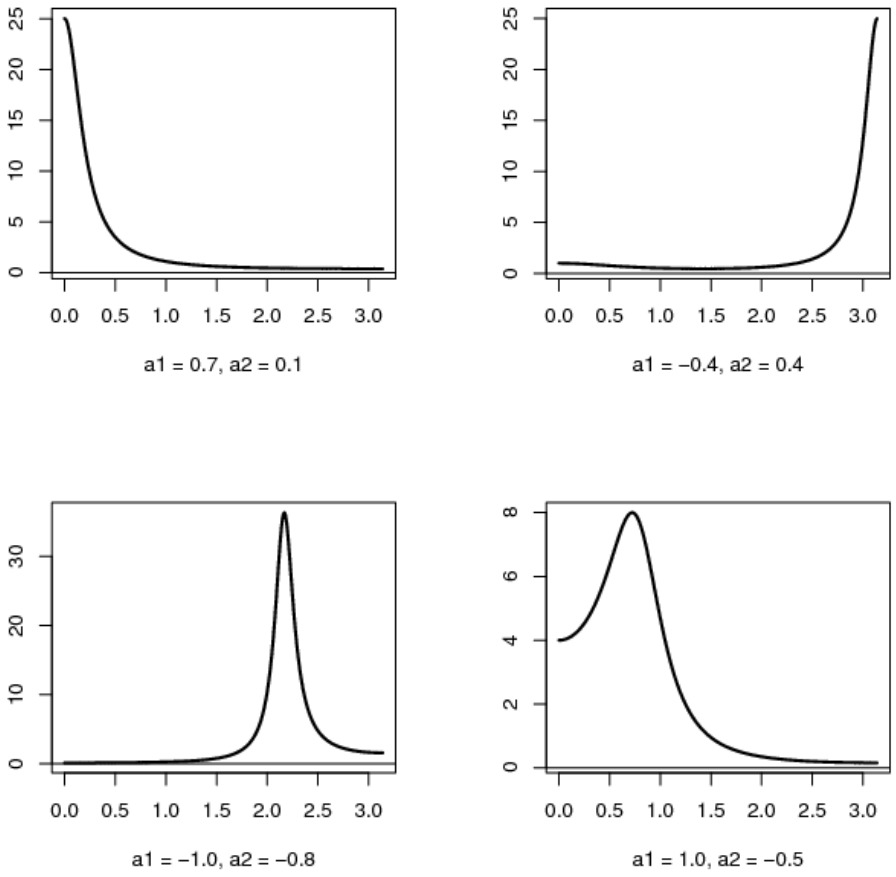

Figure 4.6 AR(2) Spectra ($2\pi f(\lambda)$), cf. Figure 3.4

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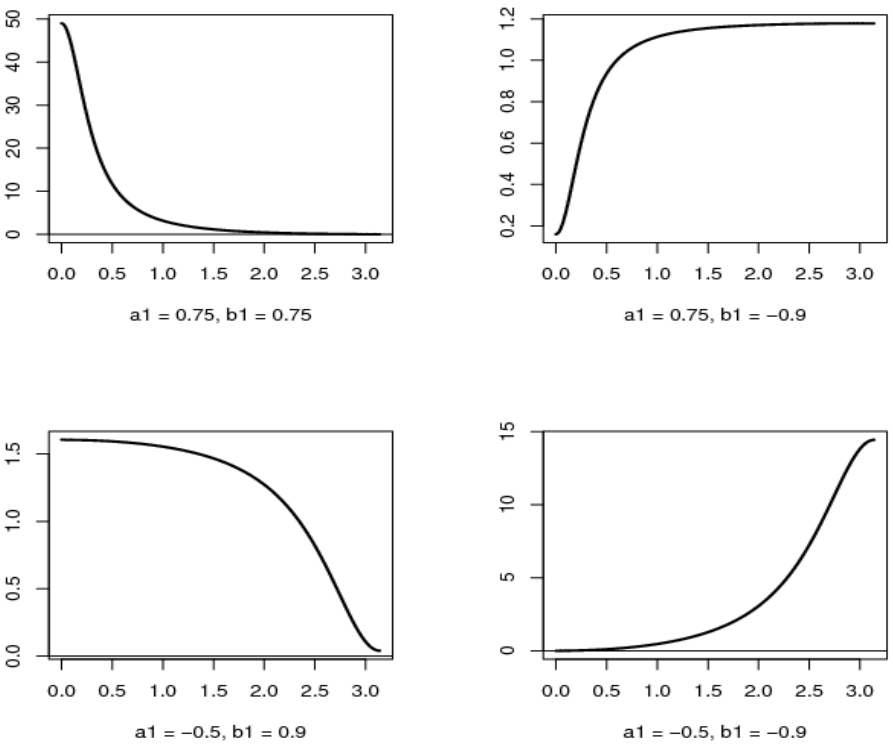

Figure 4.7 ARMA(1,1) Spectra ($2\pi f(\lambda)$), cf. Figure 3.5

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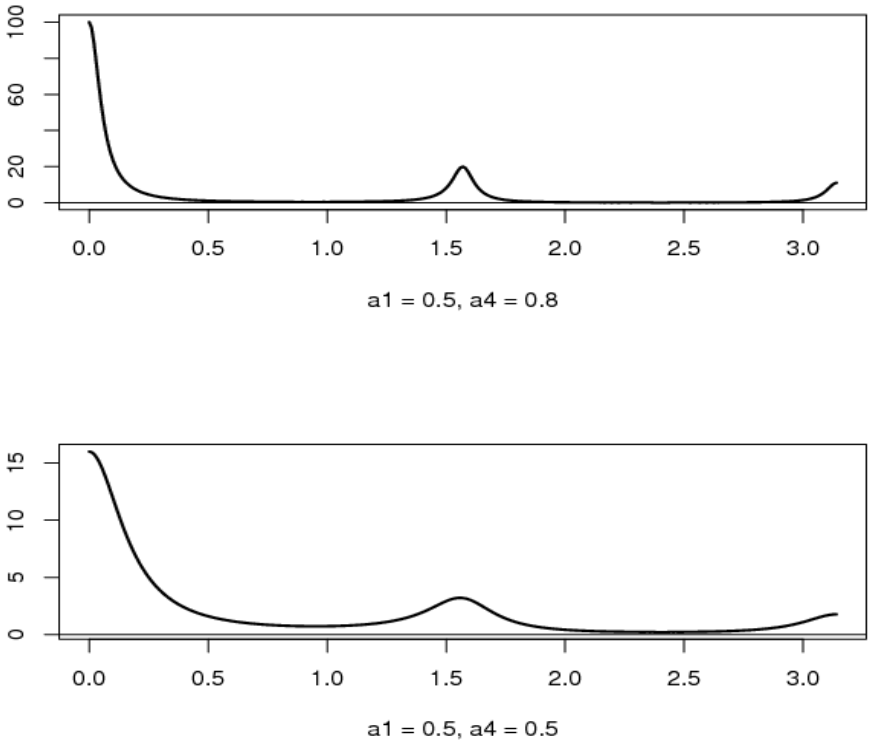

Figure 4.8 Spectra ($2\pi f(\lambda)$) of Multiplicative Seasonal AR Processess ($S = 4$)

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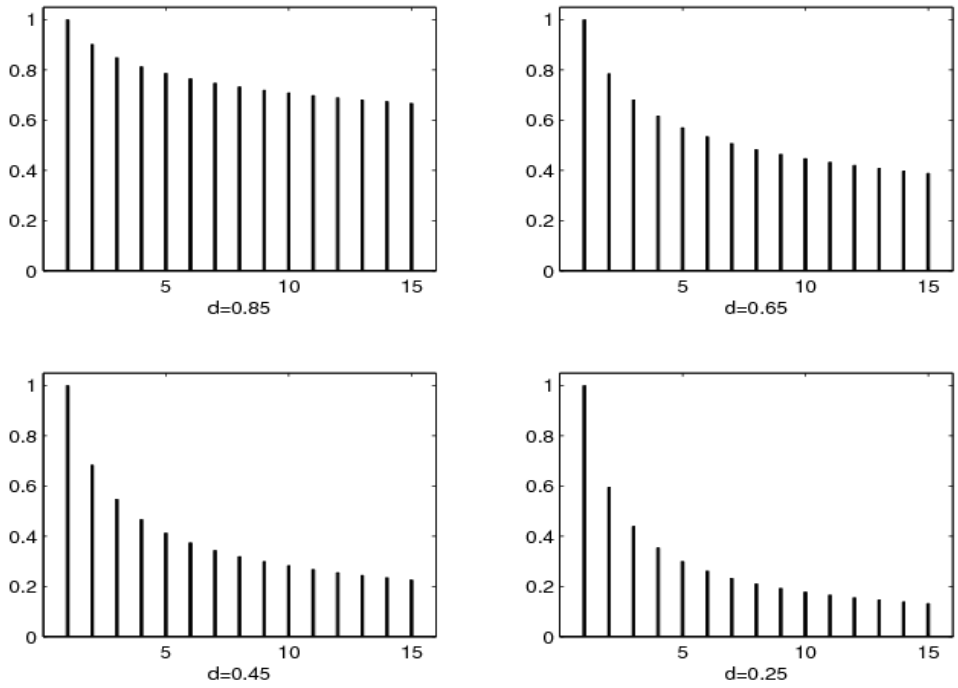

Figure 5.1 j^{d-1} for $d = 0.85, 0.65, 0.45, 0.25$

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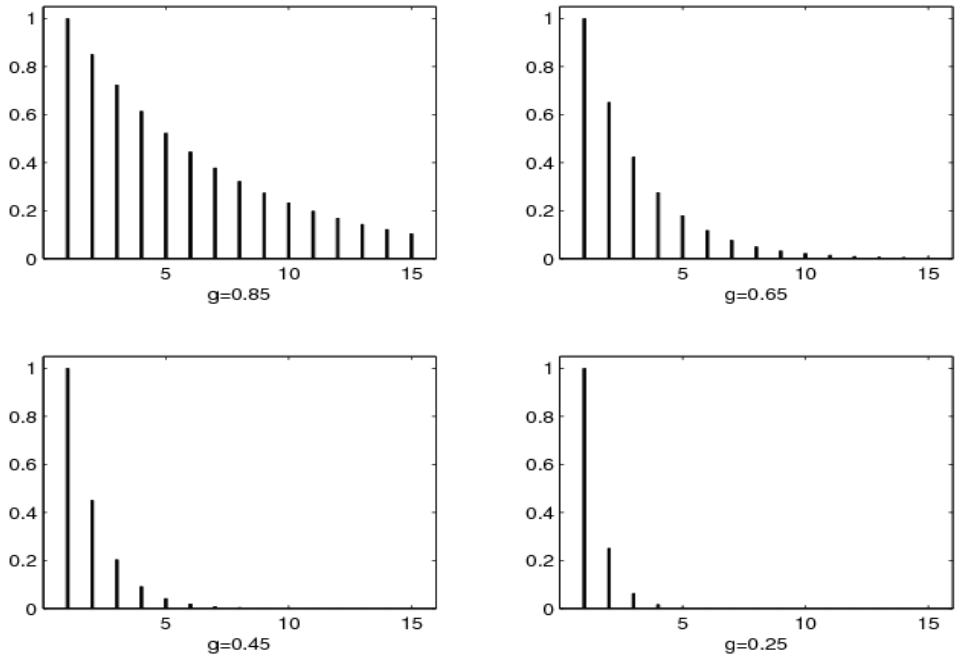

Figure 5.2 g^{j-1} for $g = 0.85, 0.65, 0.45, 0.25$

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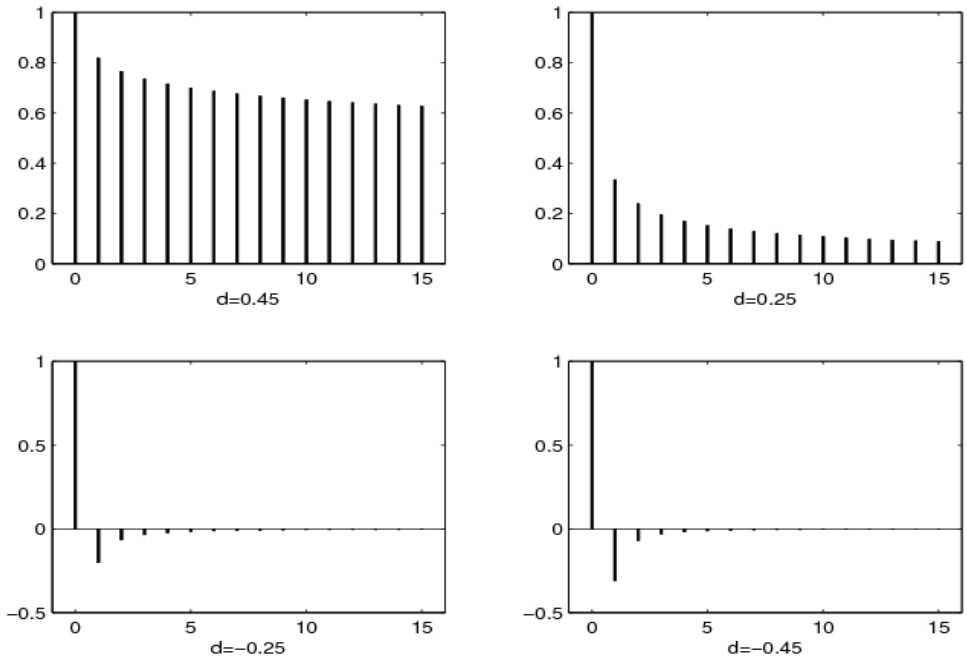

Figure 5.3 *ρ(h)* from Prop. 5.1 for d = 0.45, 0.25, -0.25, -0.45

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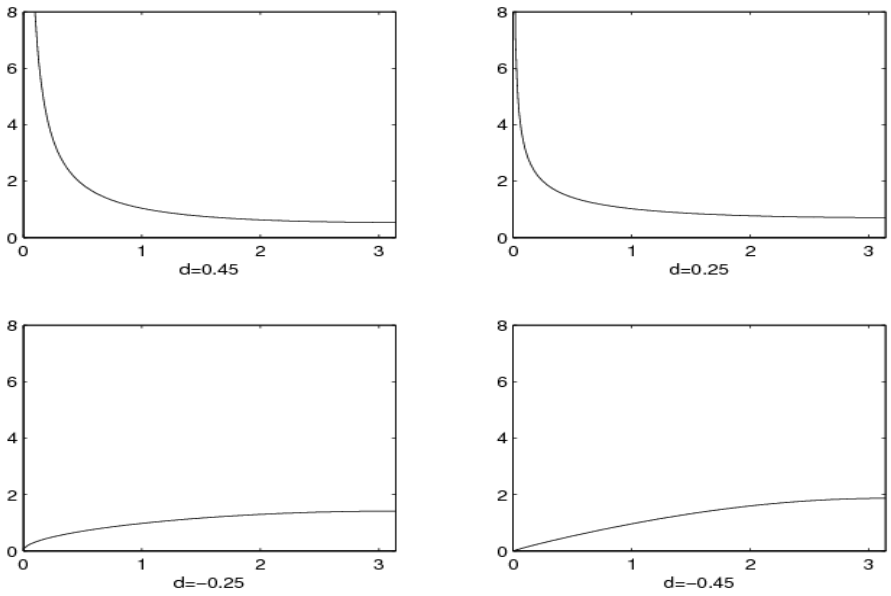

Figure 5.4 $2\pi f(\lambda)$ from Prop. 5.2 for $d = 0.45, 0.25, -0.25, -0.45$

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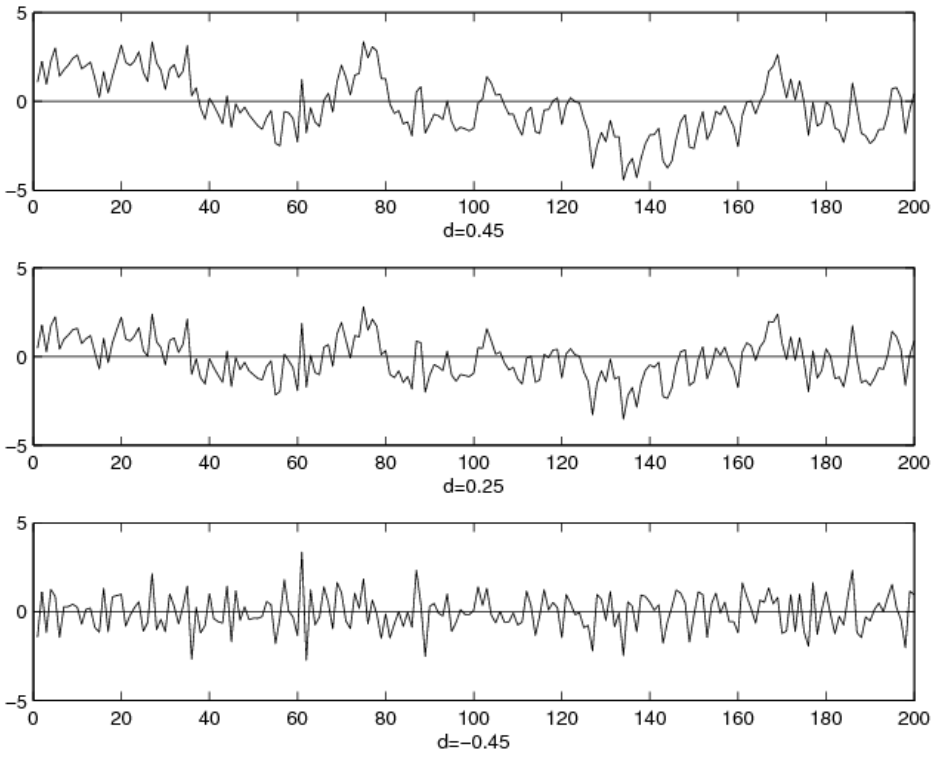

Figure 5.5 Simulated Fractional Noise for $d = 0.45, 0.25, -0.45$

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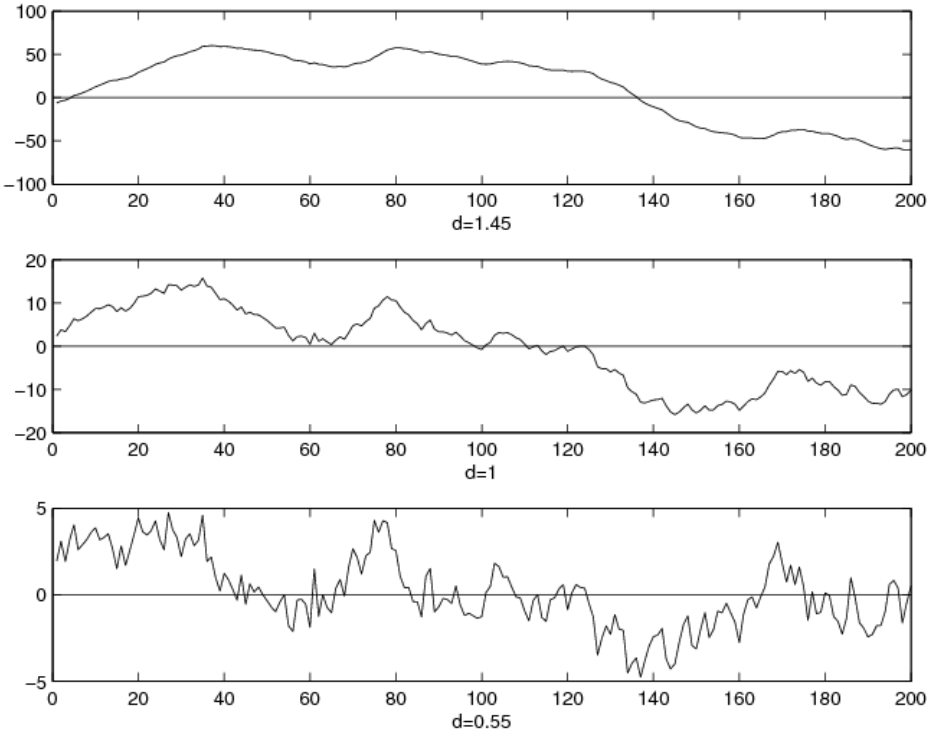

Figure 5.6 Nonstationary Fractional Noise for $d = 1.45, 1.0, 0.55$

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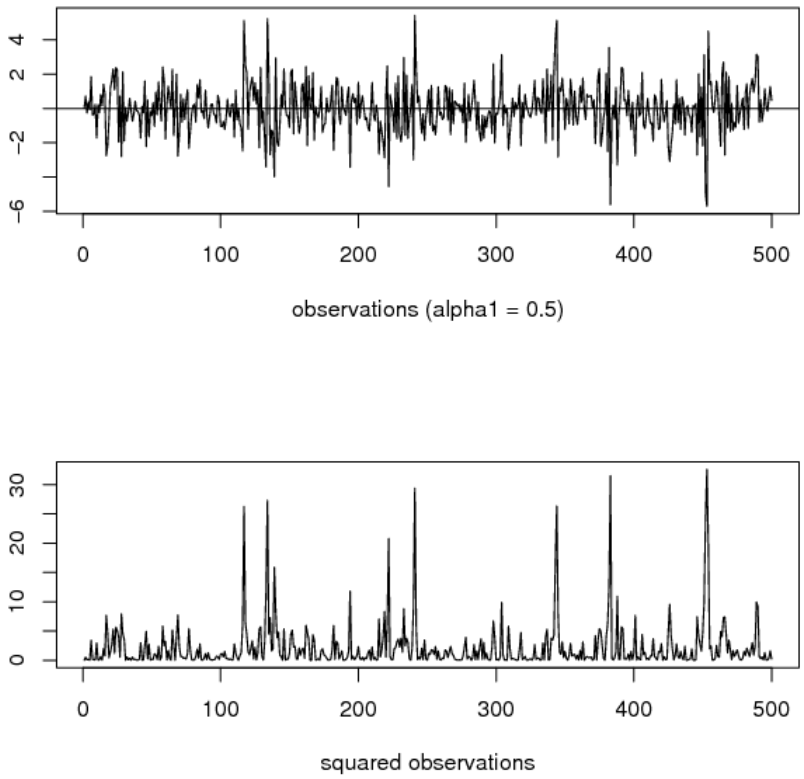

Figure 6.1 ARCH(1) with $\alpha_0 = 1$ and $\alpha_1 = 0.5$

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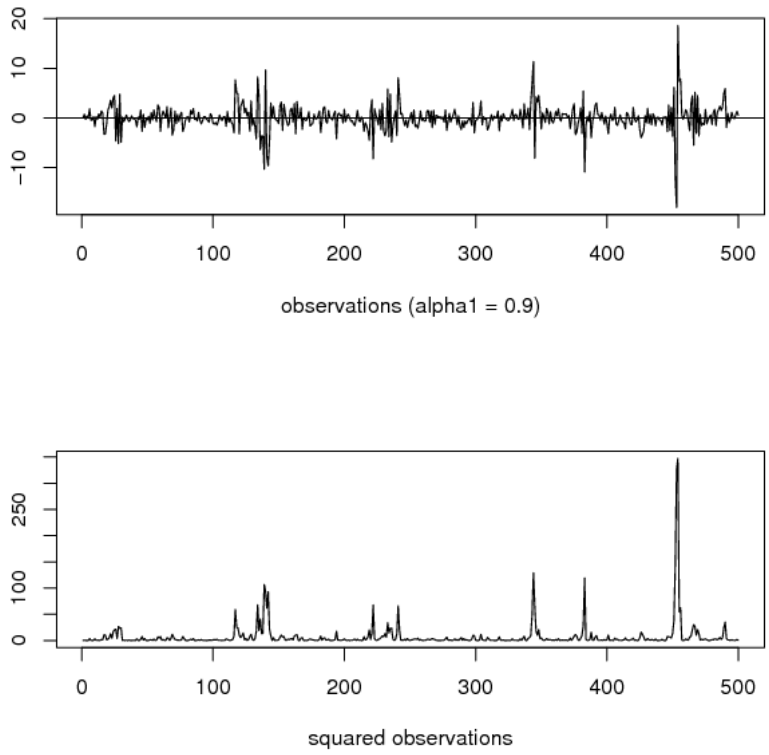

Figure 6.2 ARCH(1) with $\alpha_0 = 1$ and $\alpha_1 = 0.9$

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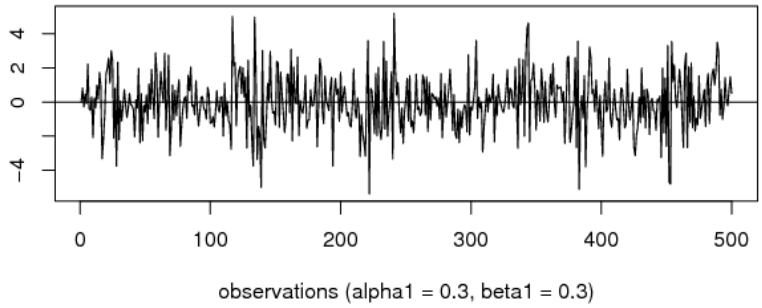

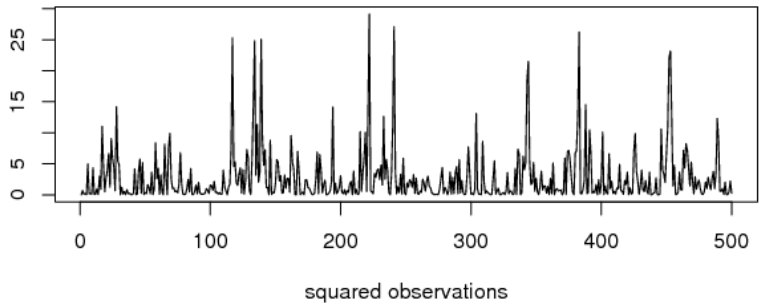

Figure 6.3 GARCH(1,1) with with $\alpha_0 = 1$ and $\alpha_1 = 0.3$ and $\beta_1 = 0.3$

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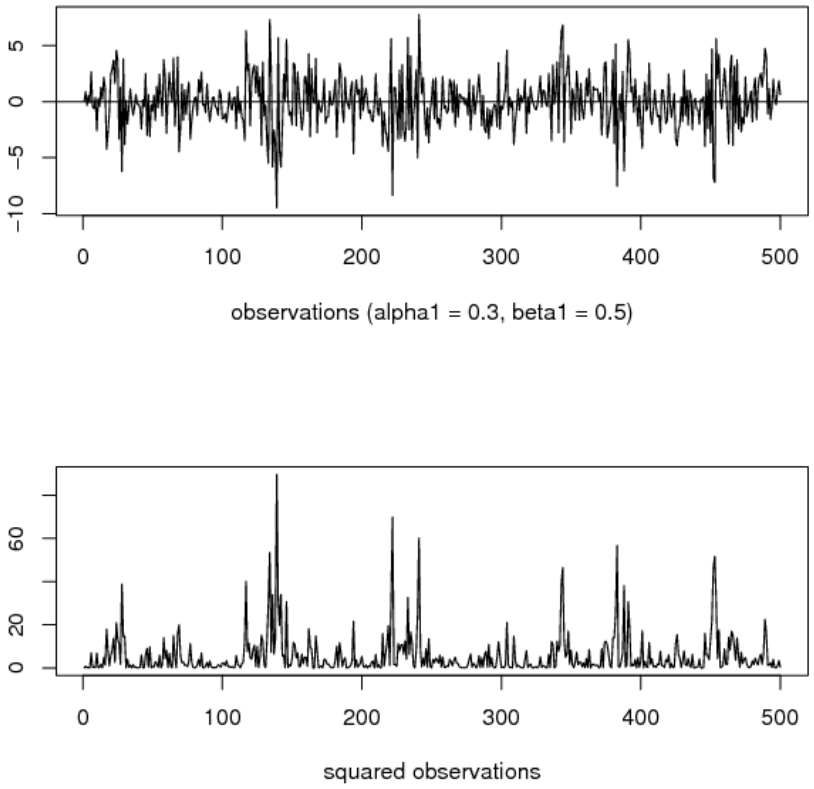

Figure 6.4 GARCH(1,1) with $\alpha_0 = 1$ and $\alpha_1 = 0.3$ and $\beta_1 = 0.5$

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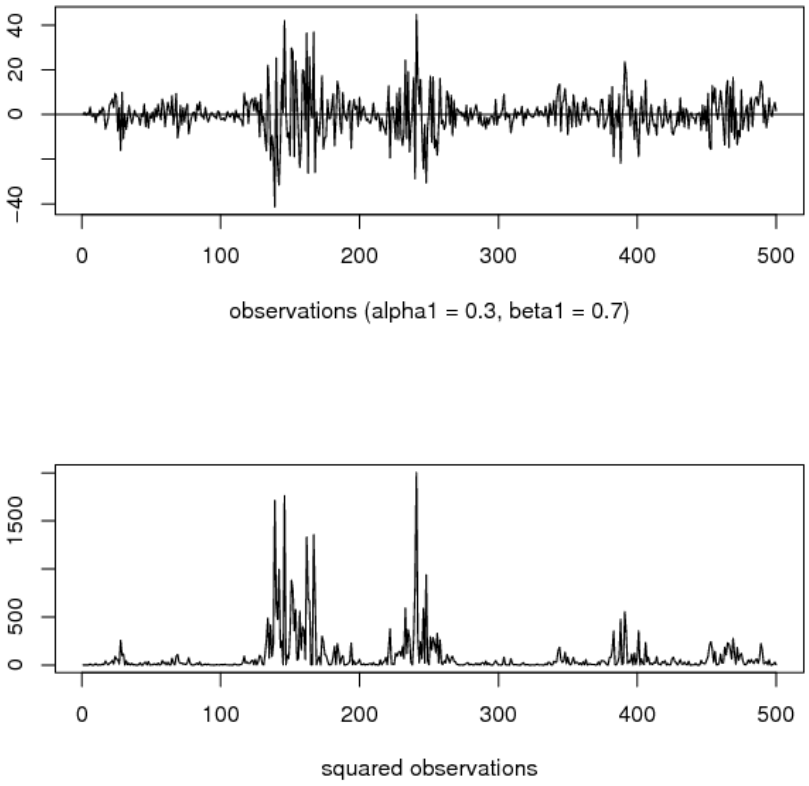

Figure 6.5 IGARCH(1,1) with $\alpha_0 = 1$ and $\alpha_1 = 0.3$ and $\beta_1 = 0.7$

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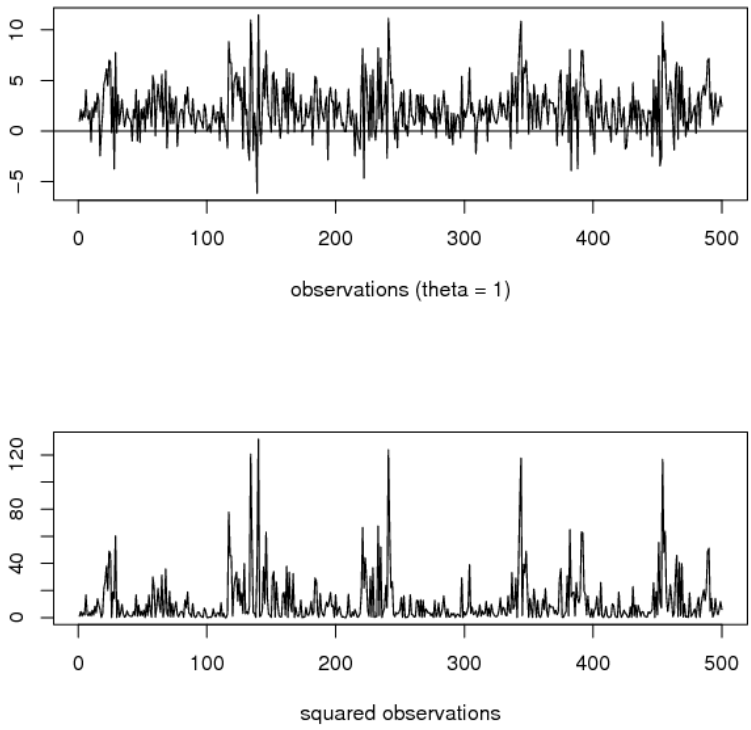

Figure 6.6 GARCH(1,1)-M from (6.11) with $\alpha_0 = 1$ and $\alpha_1 = 0.3$ and $\beta_1 = 0.5$

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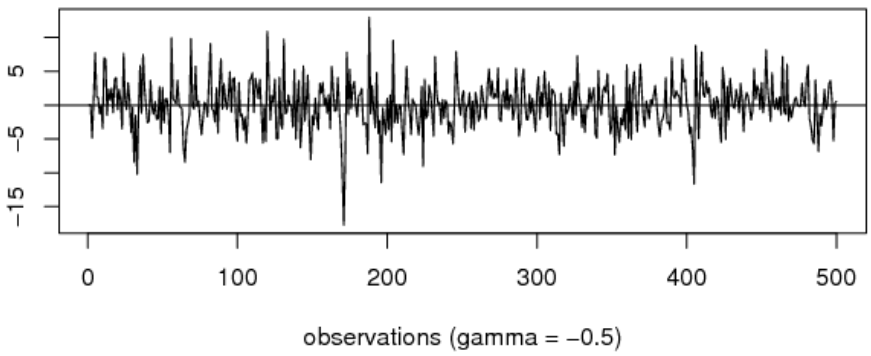

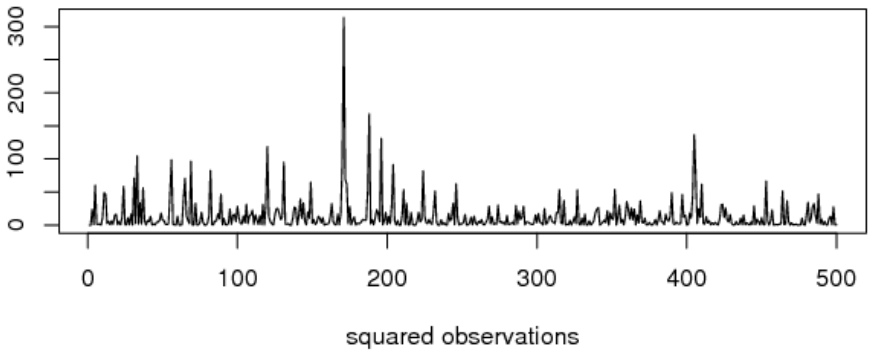

Figure 6.7 EGARCH(1,1) with $\omega = 1$, $\alpha_1 = 0.3$ and $\beta_1 = 0.5$ and $\gamma_1 = -0.5$

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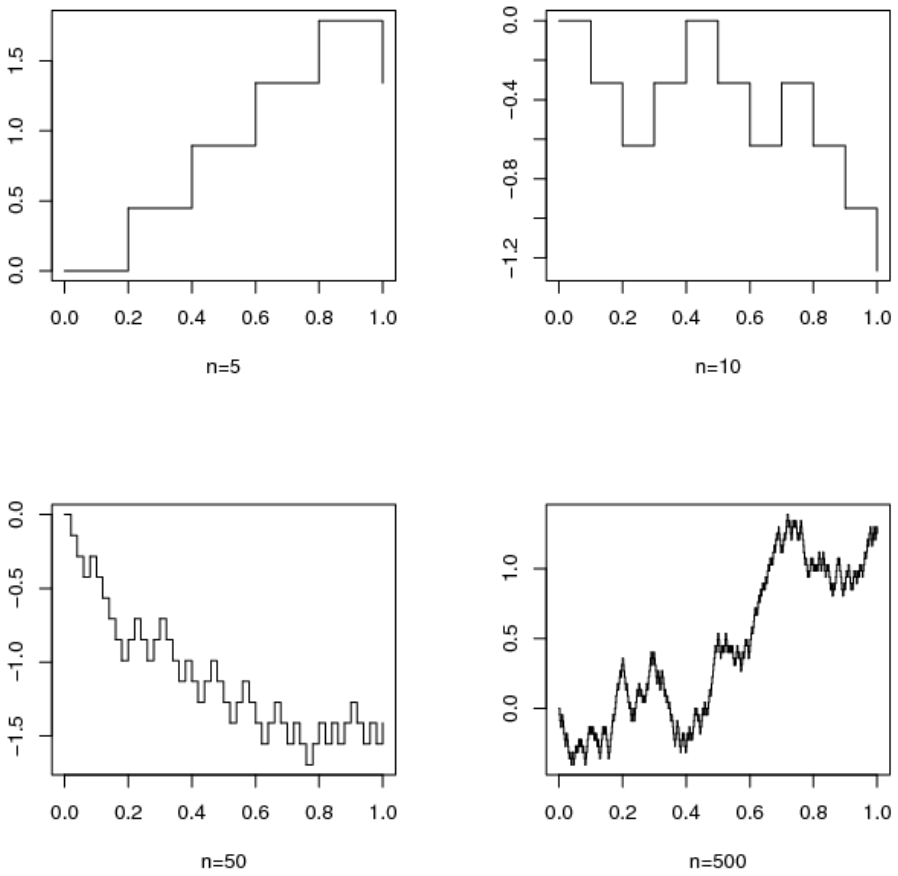

Figure 7.1 Step Function from (7.1) on the Interval [0,1]

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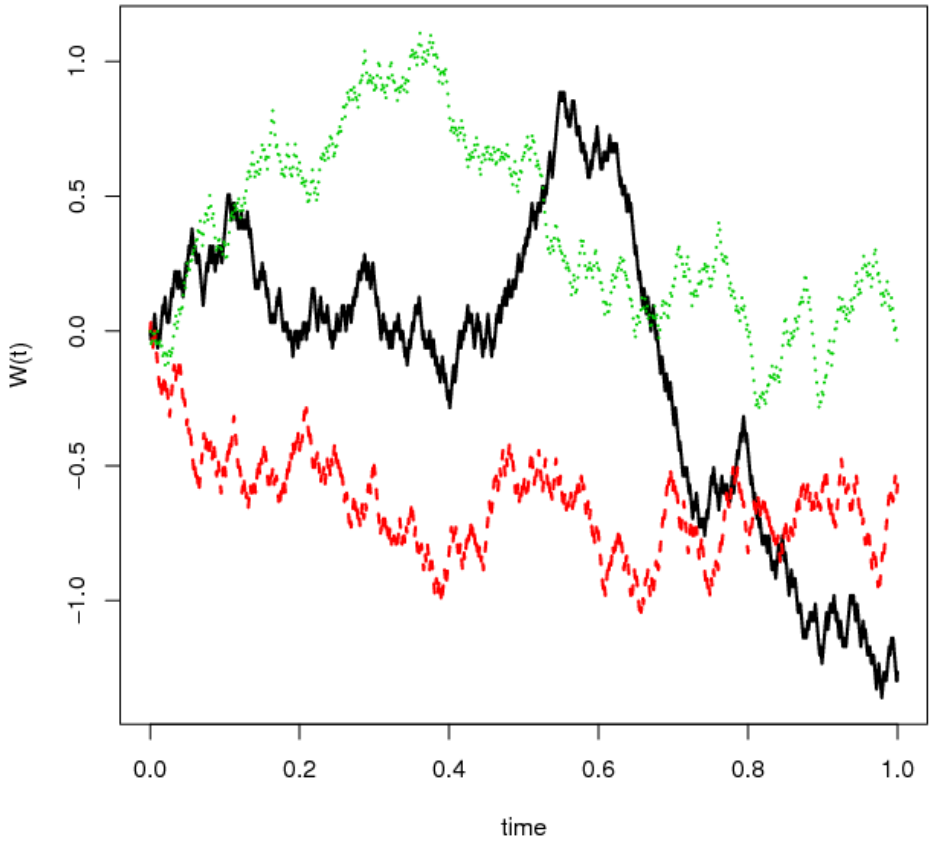

Figure 7.2 Simulated Paths of the WP on the Interval [0,1]

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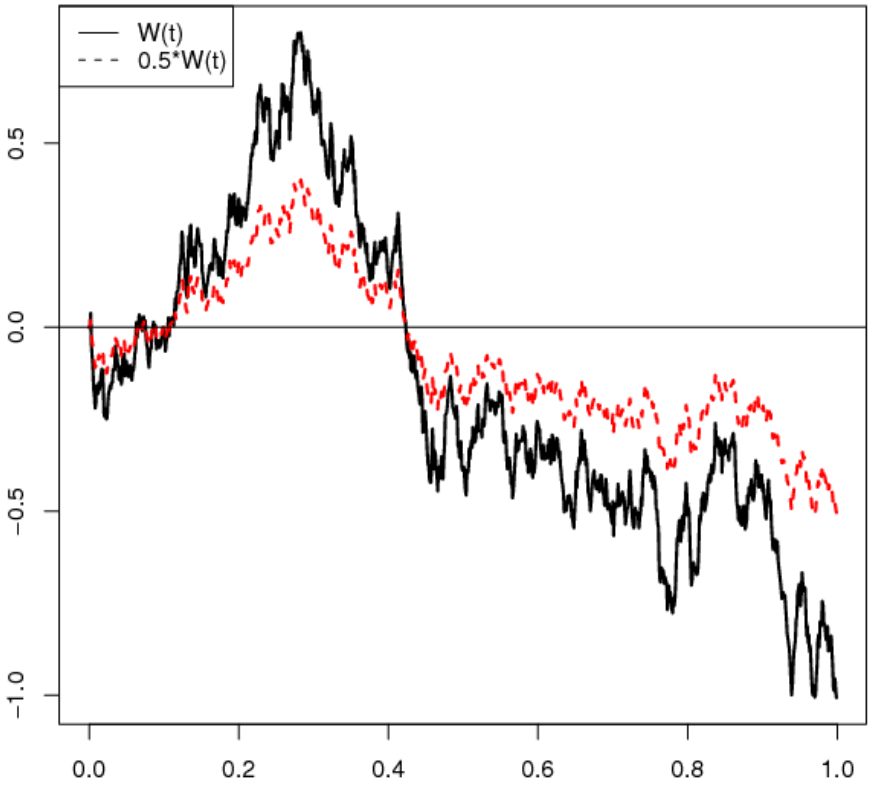

Figure 7.3 WP and Brownian Motion with σ = 0.5

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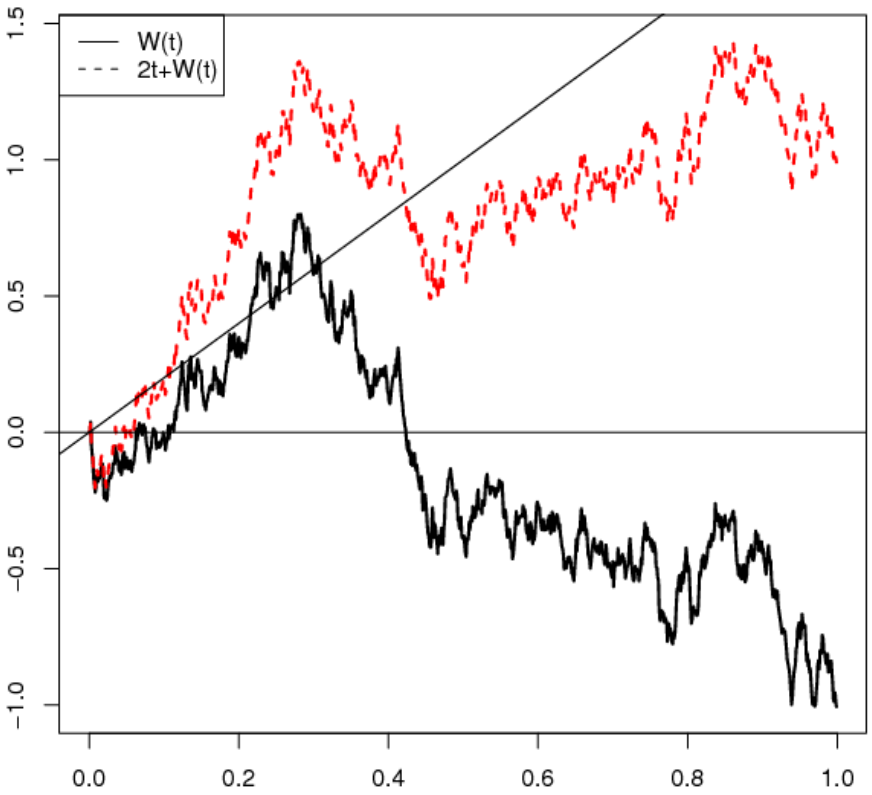

Figure 7.4 WP and Brownian Motion with Drift, where σ = 1

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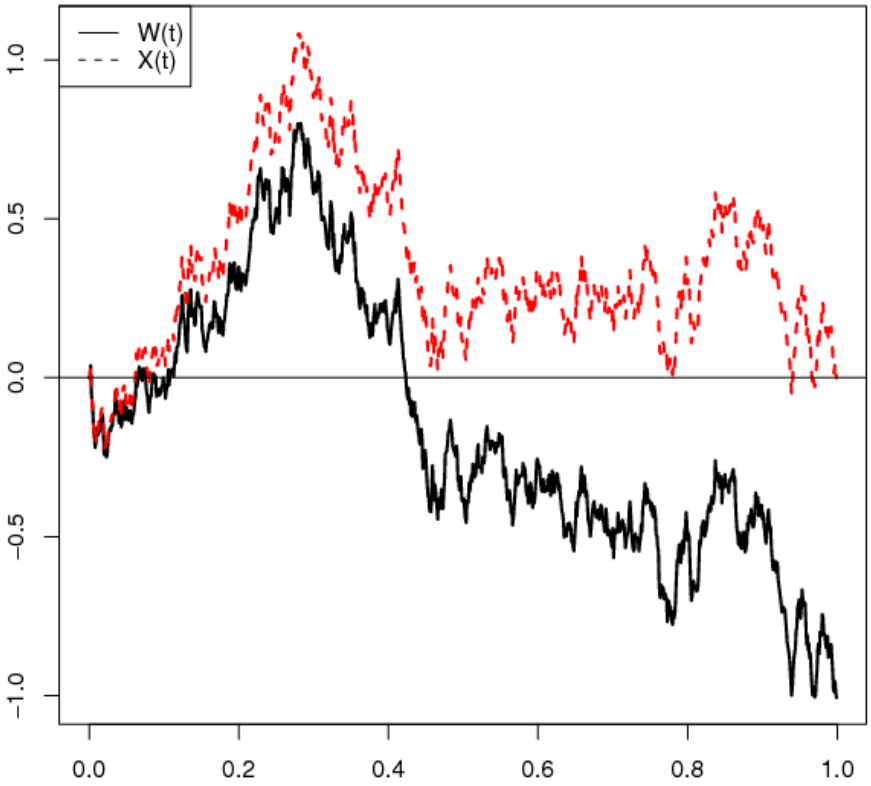

Figure 7.5 WP and Brownian Bridge ($\sigma = 1$)

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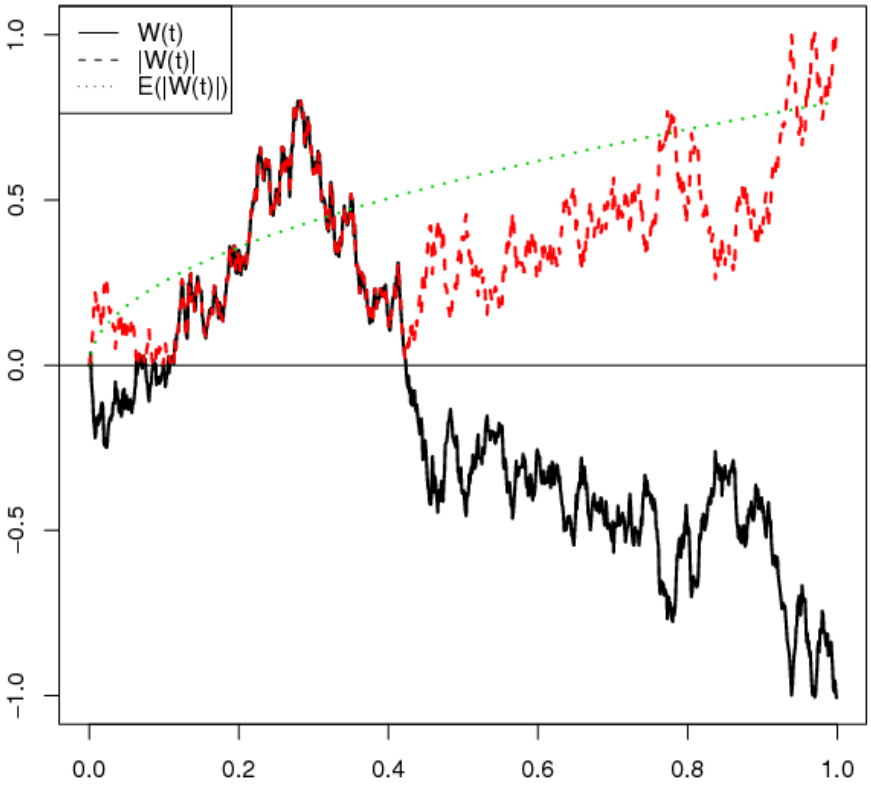

Figure 7.6 WP and reflected WP along with Expectation

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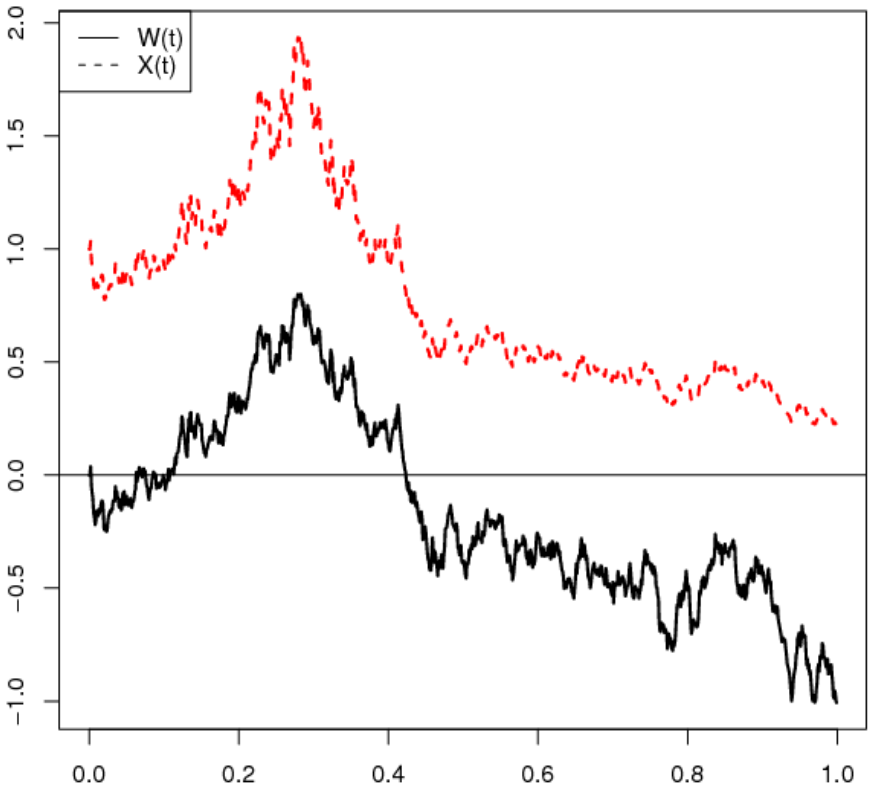

Figure 7.7 WP and Geometric Brownian Motion with $\mu = -0.5$ and $\sigma = 1$

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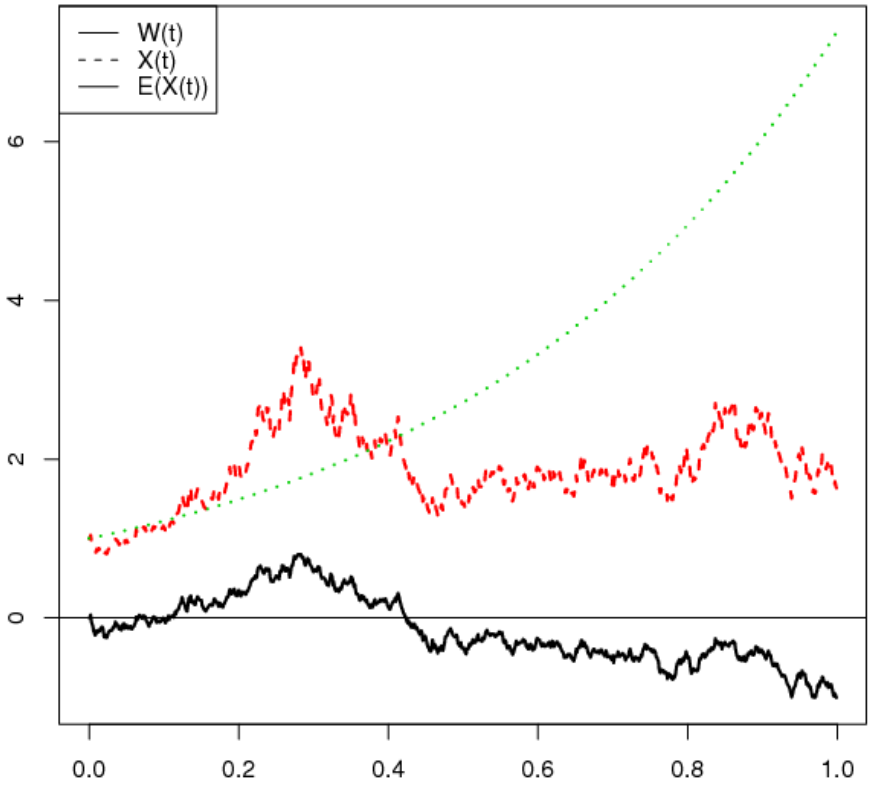

Figure 7.8 Geometric Brownian Motion with $\mu = 1.5$ and $\sigma = 1$ along with Expectation

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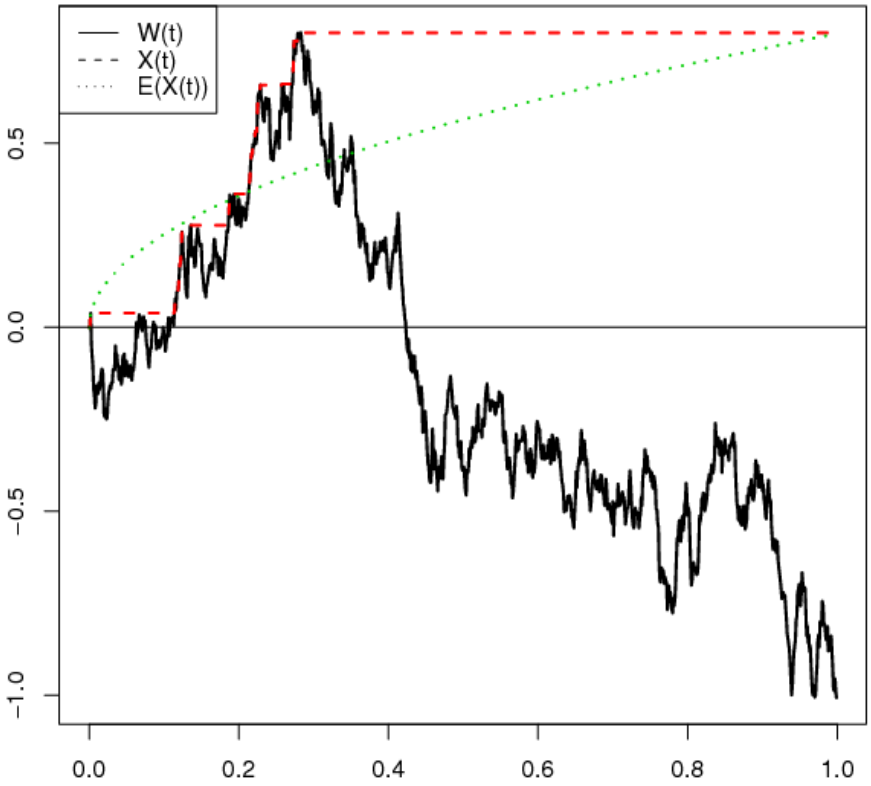

Figure 7.9 WP and Maximum Process along with Expectation

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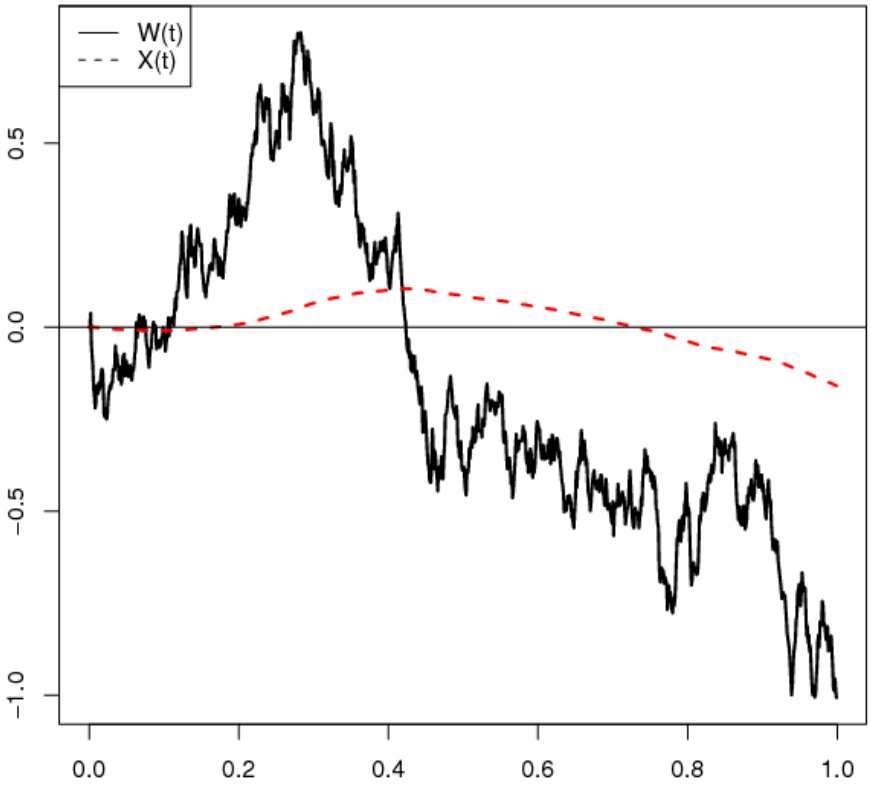

Figure 7.10 WP and integrated WP

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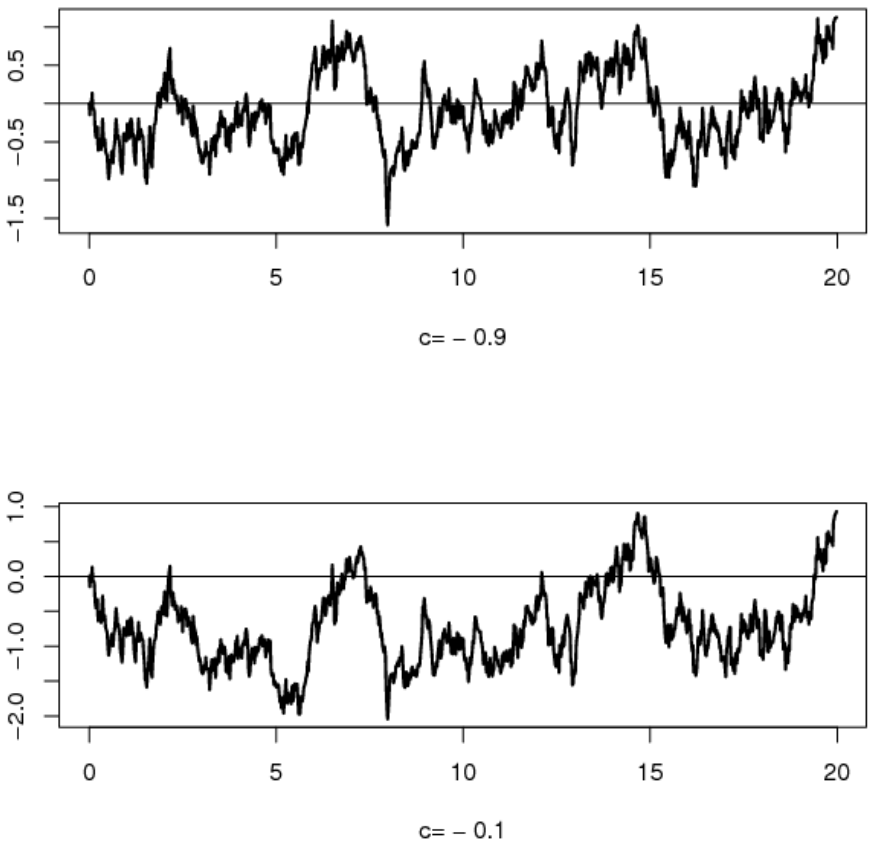

Figure 9.1 Standard Ornstein-Uhlenbeck Processes

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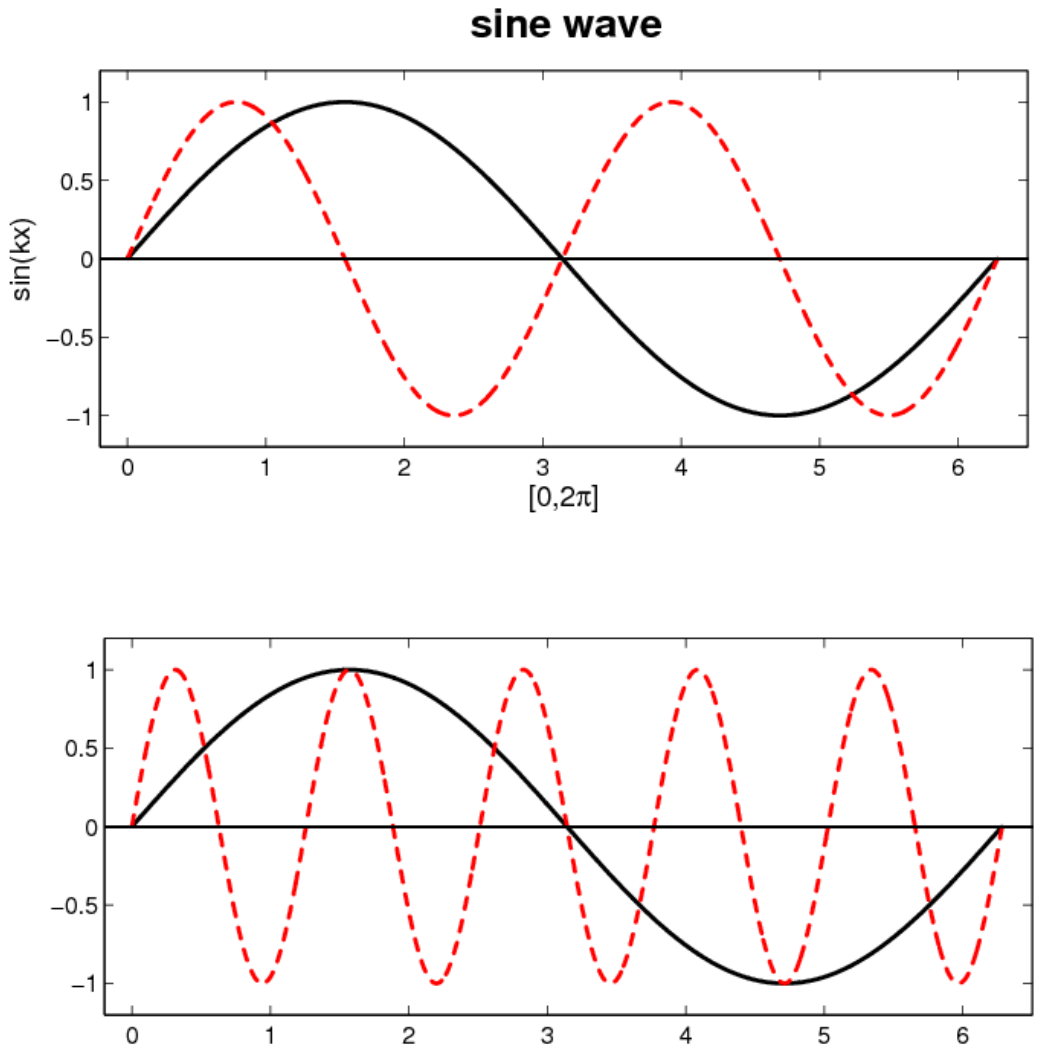

Figure 10.1 Sine Cycles of Different Frequencies (Example 10.5)

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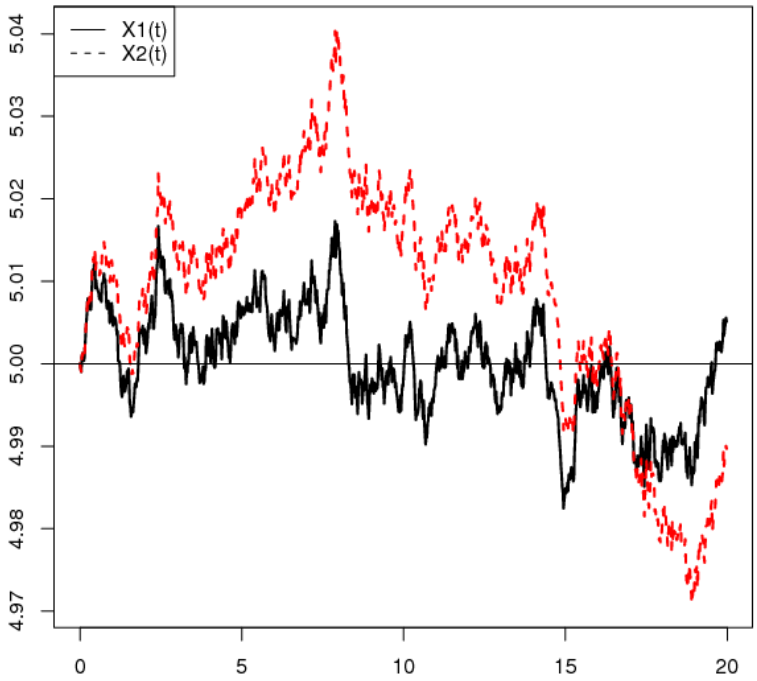

Figure 13.1 OUP for $c_1 = -0.9$ (X_1) and $c_1 = -0.1$ (X_2) ($X(0) = \mu = 5$, $\sigma_2 = 0.01$)

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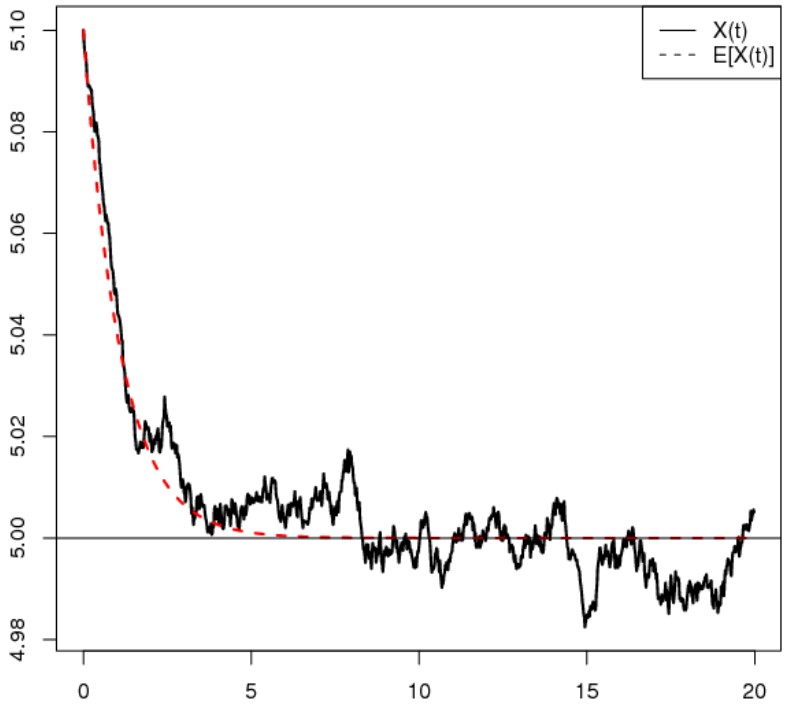

Figure 13.2 OUP for $c_1 = -0.9$ and Starting Value $X(0) = 5.1$ including Expected Value Function ($\mu = 5$, $\sigma_2 = 0.01$)

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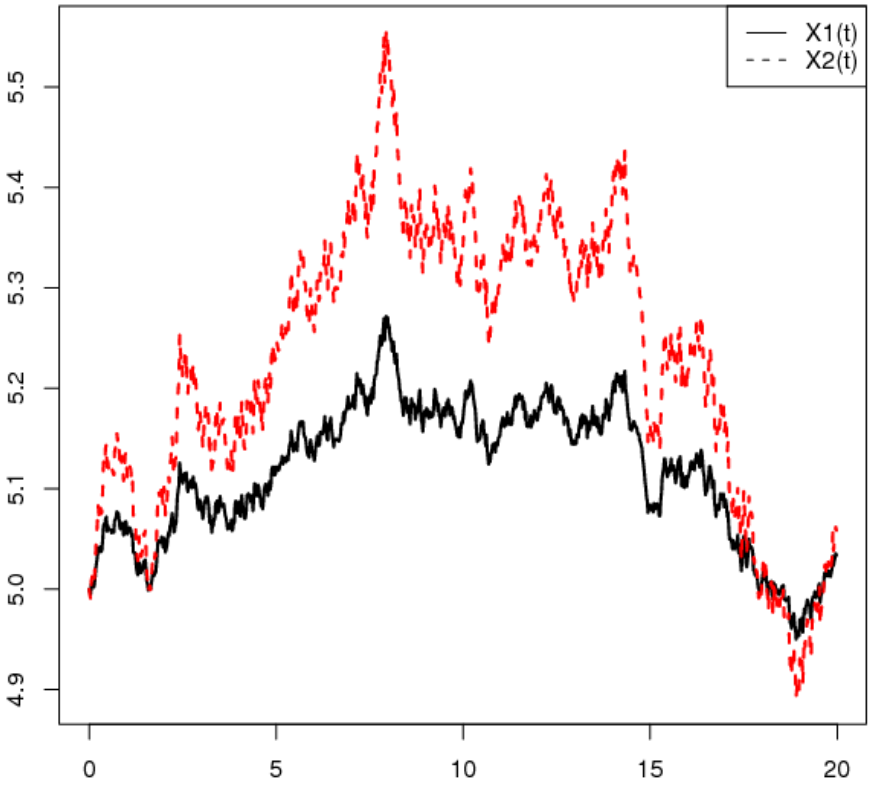

Figure 13.3 Dothan for $\sigma_1 = 0.01$ (X_1) and $\sigma_1 = 0.02$ (X_2) ($X(0) = \mu = 5$)

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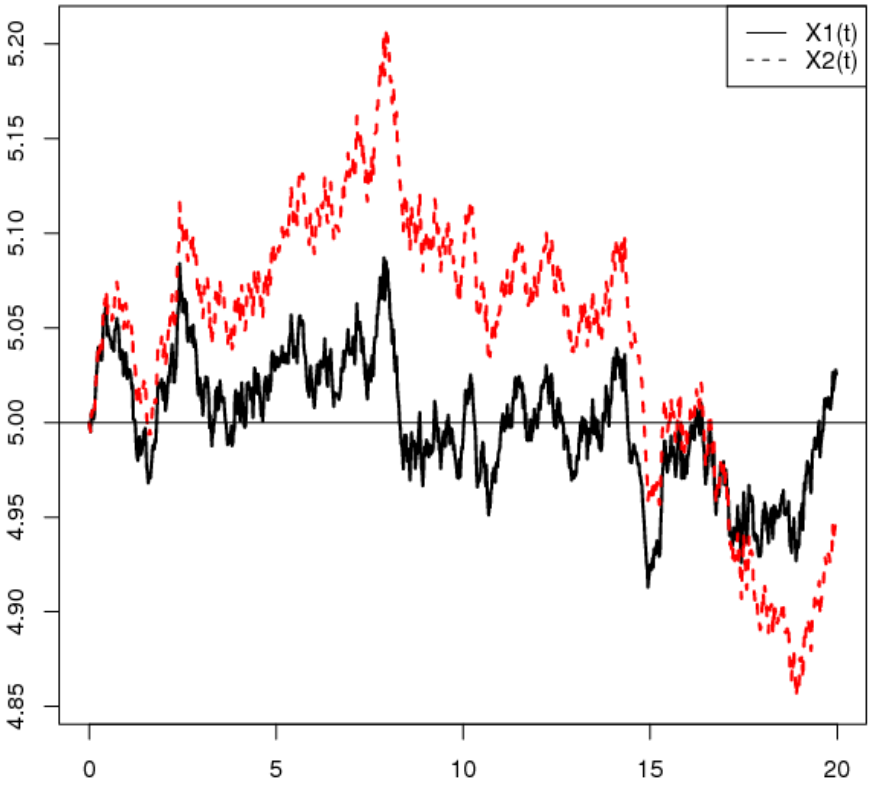

Figure 13.4 Brennan-Schwartz for $c_1 = -0.9$ (X_1) and $c_1 = -0.1$ (X_2) ($X(0) = \mu = 5$, $\sigma_1 = 0.01$)

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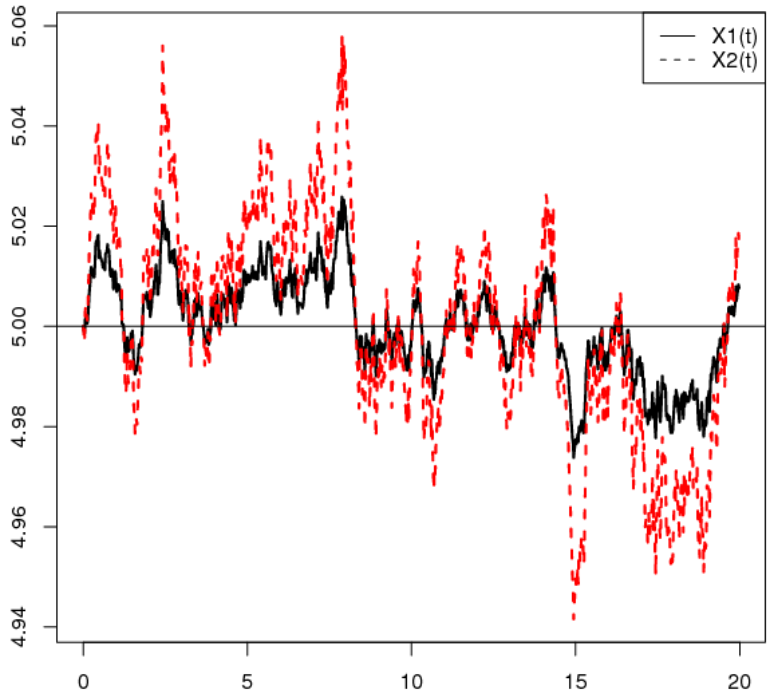

Figure 13.5 CKLS with $\gamma = 0.25$ (X_1) and $\gamma = 0.75$ (X_2) for $c_1 = -0.9$ ($X(0) = \mu = 5$, $\sigma_1 = 0.01$)

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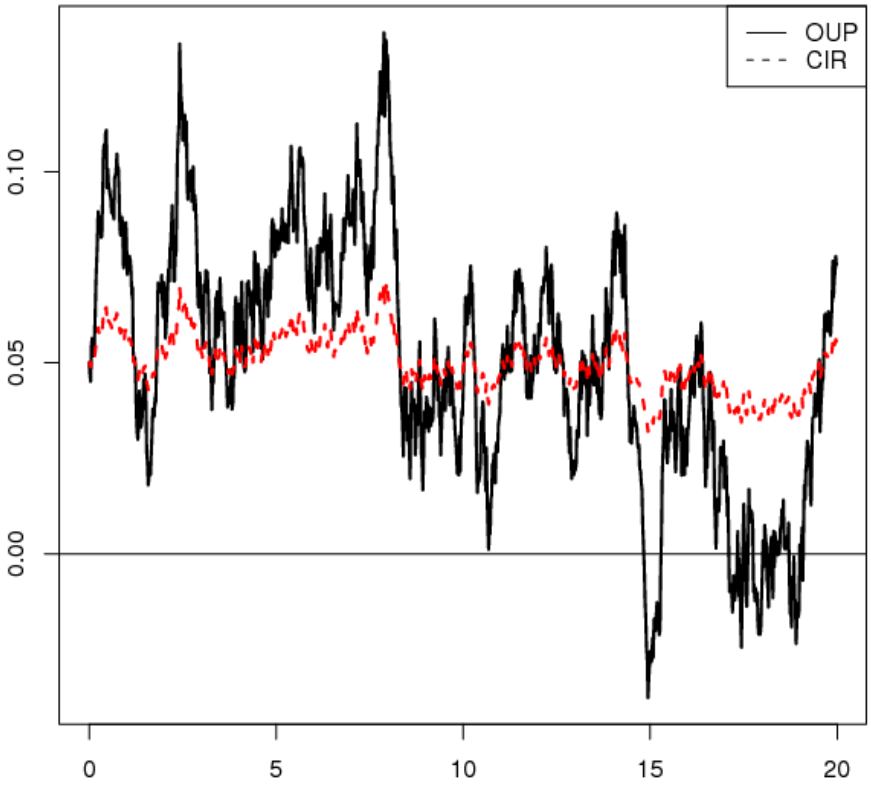

Figure 13.6 OUP and CIR for $c_1 = -0.9$ ($X(0) = \mu = 5$, $\sigma = \sigma_2 = 0.01$)

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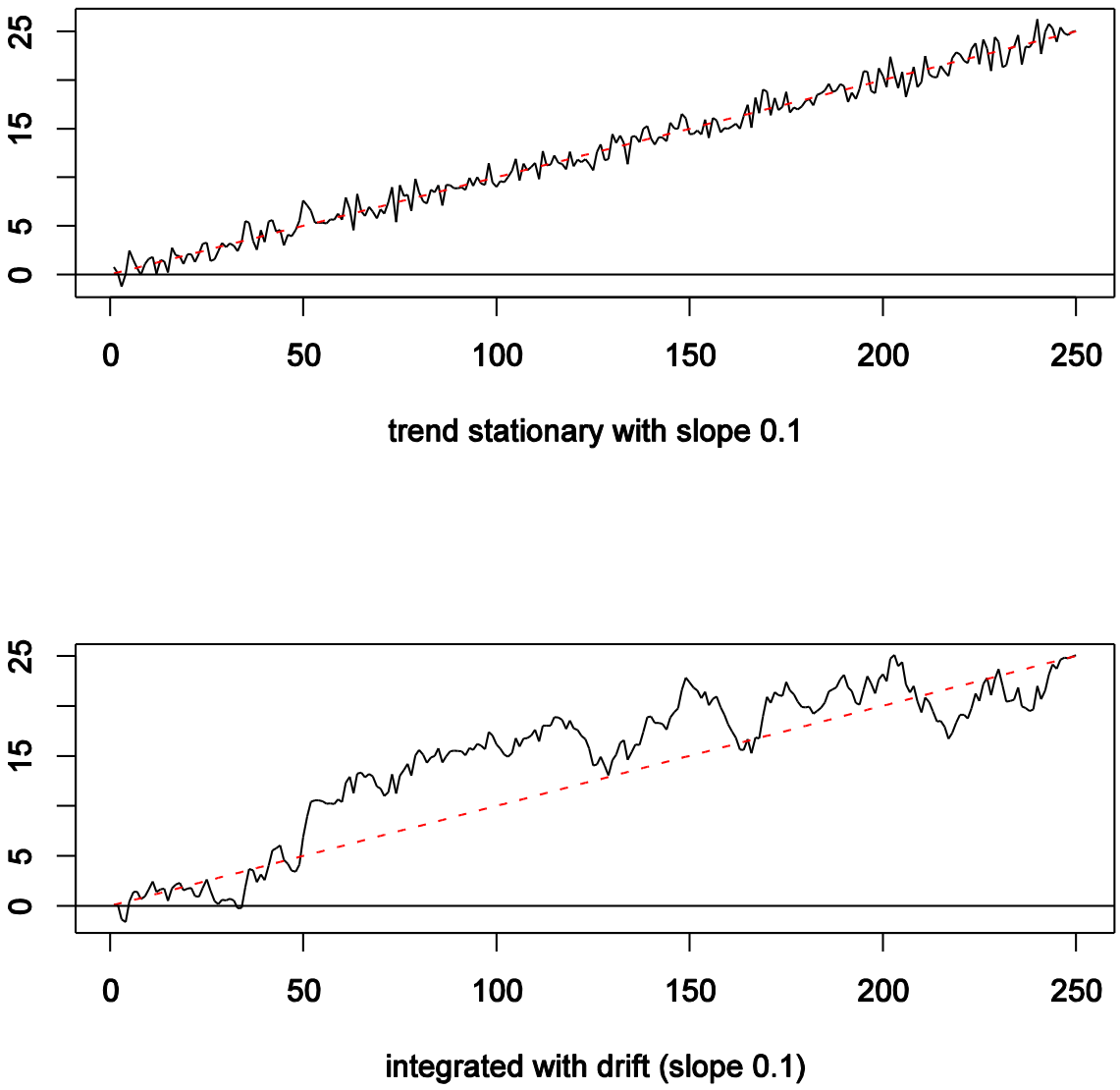

Figure 15.1 Linear Time Trend

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